

Black Holes

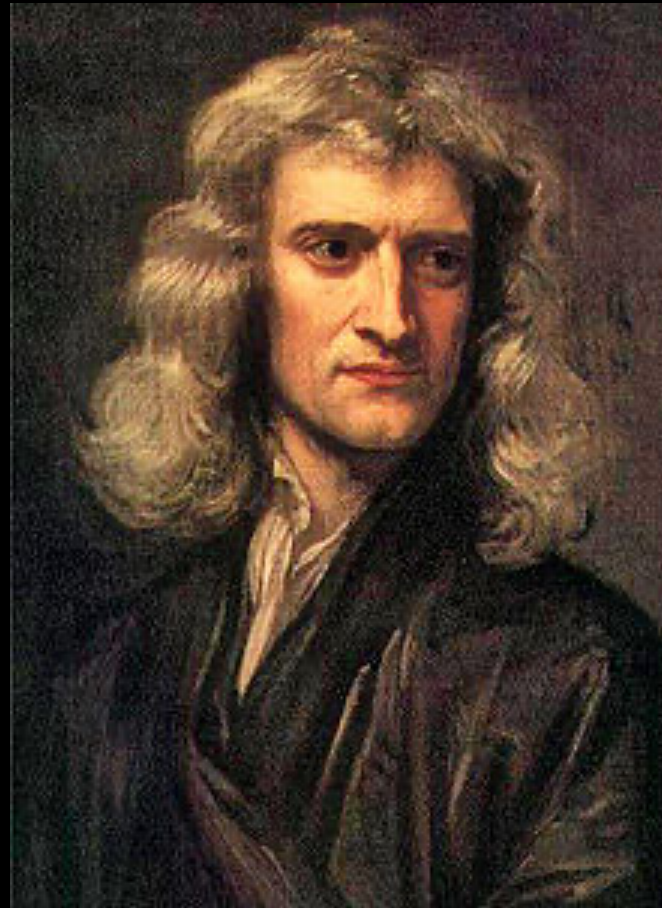
by

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BSc (Physics), PhD (Physics), PhD (Psychology)

Sir Isaac Newton

(1642 - 1727)



Newton

- Introduced the concept of Gravity
- Established the mathematical relationship between Forces and Masses
- Showed that these forces were not simply local but universal

To launch a satellite

- Do not send it straight up!
- Isaac Newton's cannon ball!
- To leave the earth
- Must achieve escape velocity

Escape Velocity

- The minimum speed needed for an object to "break free" from the gravitational attraction of a massive body
- From Earth: about 40,270 km/h (25,020 mph)
- When an object's Kinetic Energy is equal to its Gravitational Potential Energy

Kinetic Energy = Potential Energy

$$\frac{1}{2} m v^2 = \frac{G M m}{r}$$

Escape Velocity

(40,270 km/h)

$$v_e = \sqrt{\frac{2GM}{r}}$$

The Space Age

- Newton's equations are:
- Used to calculate payloads
- Used to calculate rocket trajectories

Limitations of Newtonian Mechanics

- In very large gravitational fields
- At very high velocities
- We must use Einsteinian Mechanics

Albert Einstein

(1879 - 1955)



Relativity

- Special Theory 1905
- General Theory 1915

Speed of Light

- Is constant
- In vacuum 299,792 km/sec
- Nothing can go faster

Escape Velocity

(40,270 km/h)

$$v_e = \sqrt{\frac{2GM}{r}}$$

Velocity of Light (299,792 km/sec)

$$c = \sqrt{\frac{2GM}{r}}$$

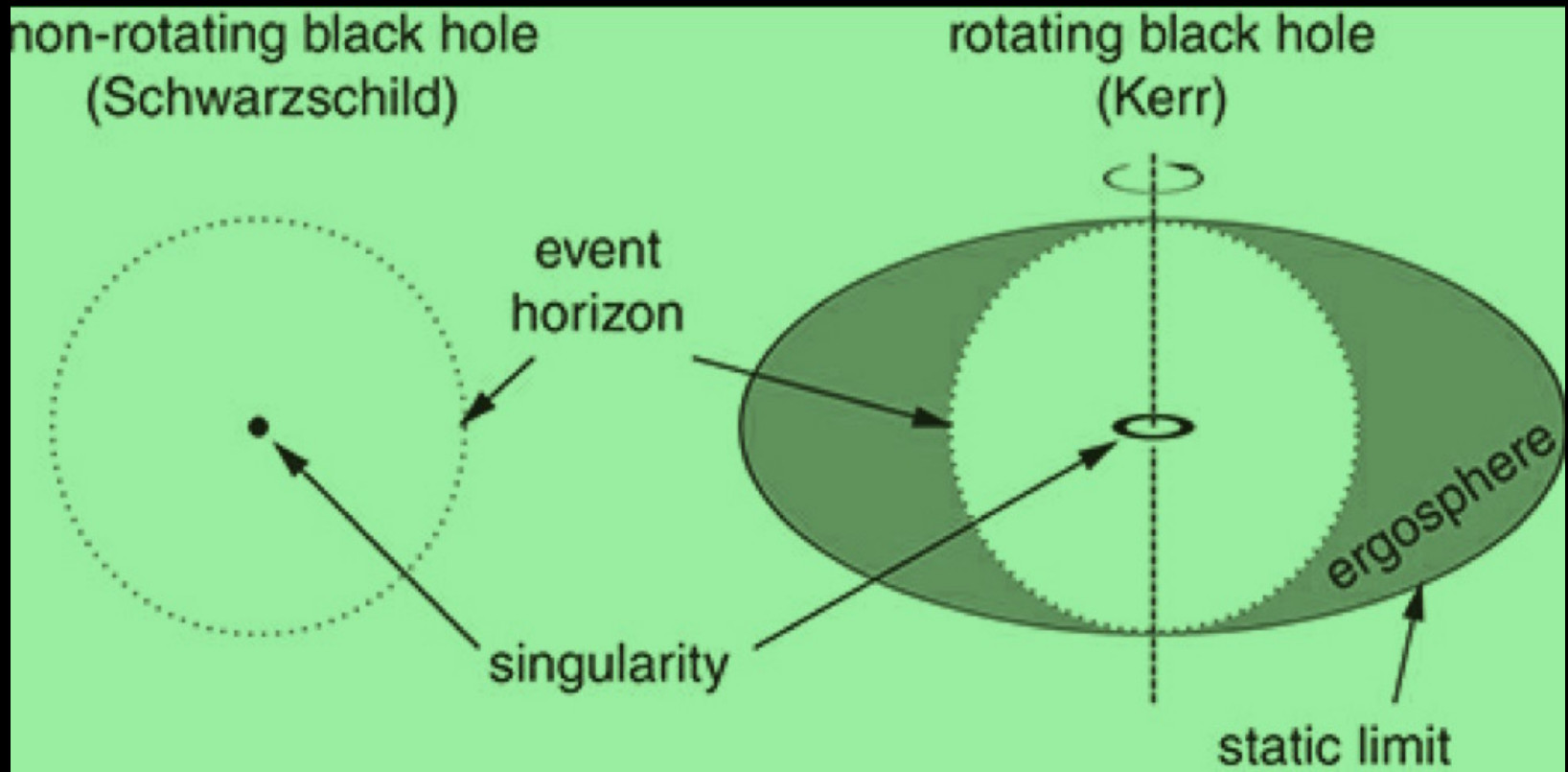
Schwarzschild Radius of a non-rotating Black Hole

$$r_s = \frac{2GM}{c^2}$$

Karl Schwarzschild (1873 – 1916)



Rotating Black Holes (Kerr Black Holes)



- In alternative terminology it is the radius of the

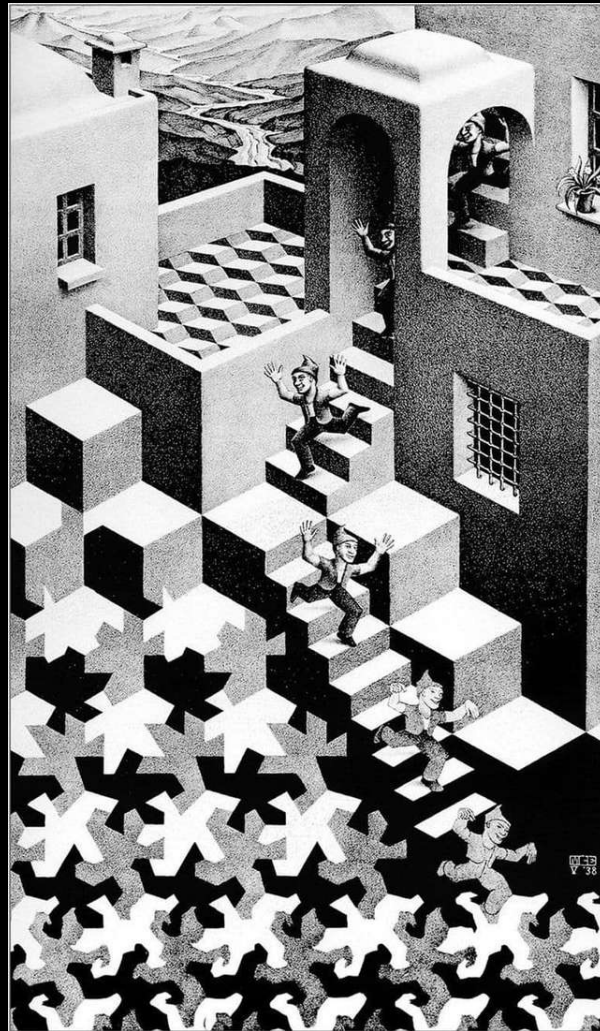
- Event Horizon

- It is very likely that ALL black holes are rotating
- It is likely that there is one at the centre of every galaxy
- There IS one at the centre of our galaxy

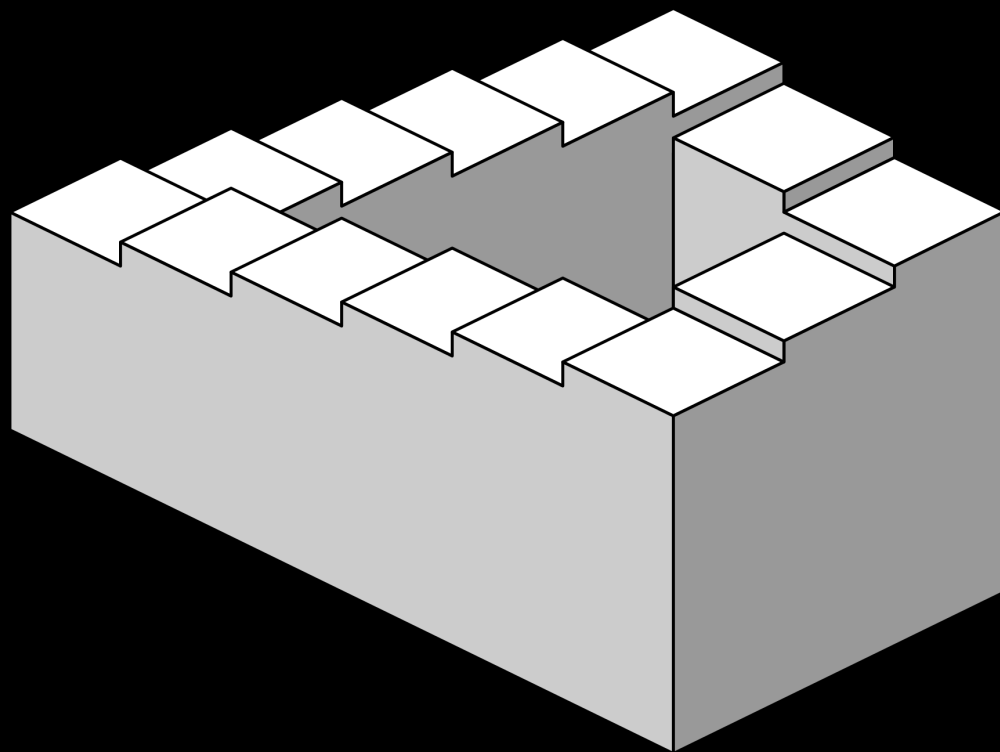
Sir Roger Penrose (1931 -)



M C Escher (1938)



Penrose Stairs (1958)



M C Escher (1960)



M C Escher (1962)



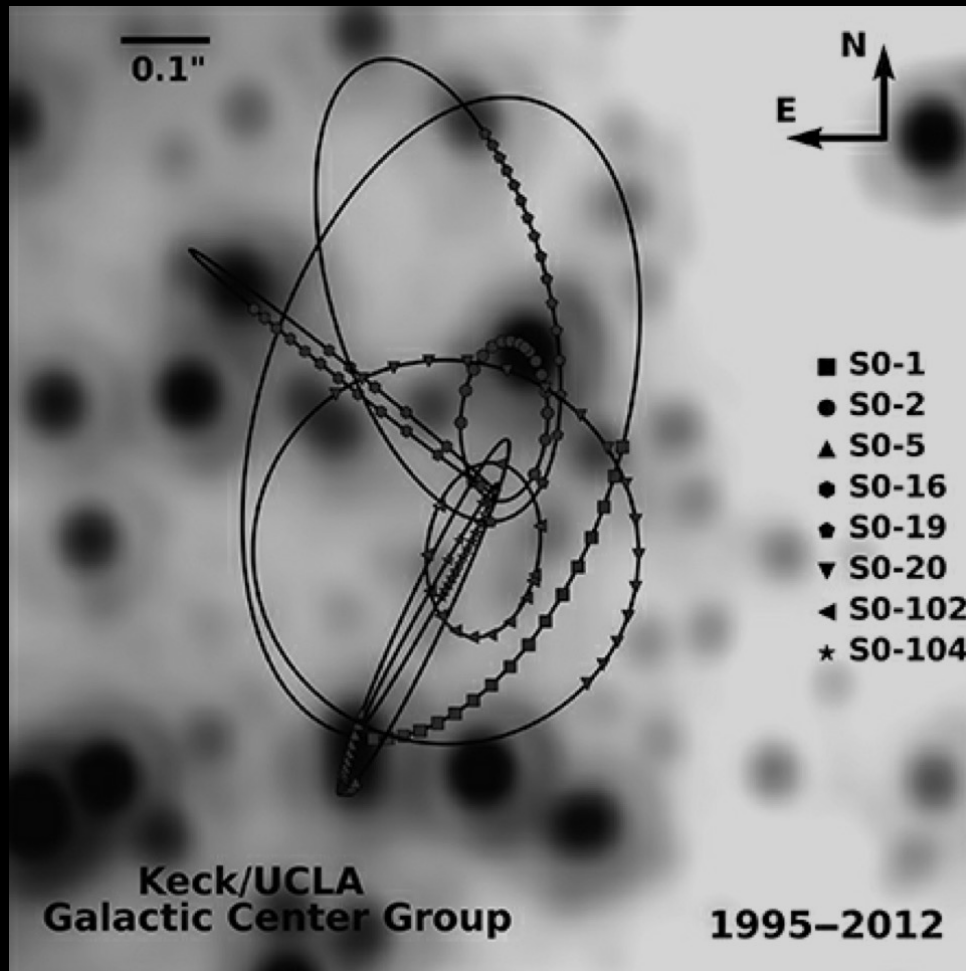
Reinhard Genzel
(1952 -)



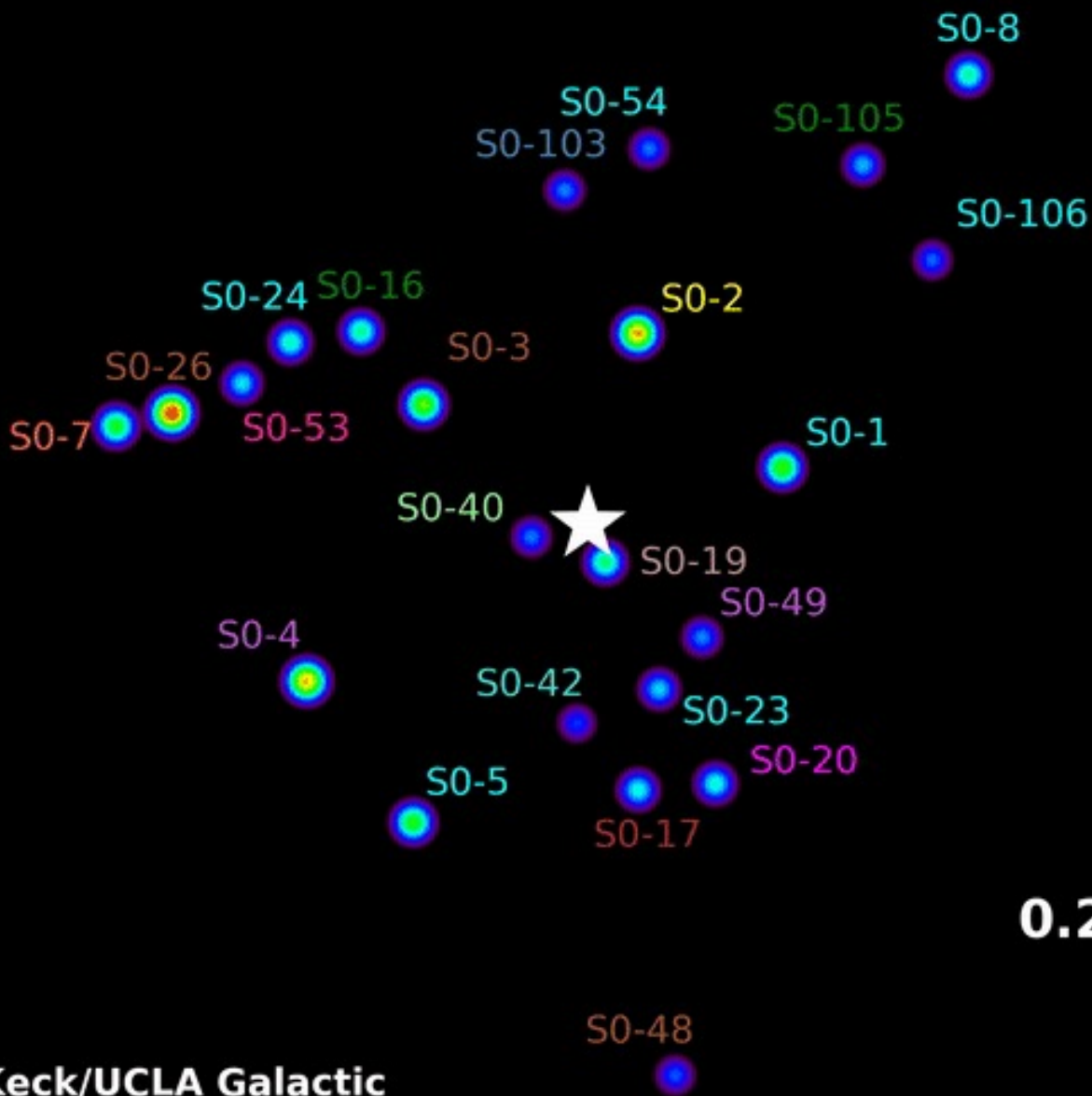
Andrea Ghez
(1965 -)



The Centre of the Milky Way



1995.5



Keck/UCLA Galactic
Center Group

The Centre of the Milky Way

- A massive Black Hole
- 4 Million times the mass of the Sun
- It has a radius less than 6 light hours

Lots of Black Holes

- They are not only at the centre of galaxies
- When stars die they *can* become black holes
- We must consider the life of a star

The Life of a Star

- Nobody lives long enough to follow the birth, the growth, the maturity and the death of a star
- Life on earth has not been long enough!
- But in the same way we can look at a forest of trees and recognise the young and the old trees, so we can with stars

The Life of a Star

- Stars form and grow by accretion
- The size to which they grow is determined by the available material
- Planets form in the accretion disc and also grow
- Frequently double stars form

The Life of a Star

- As a star grows the gravitational forces pulling it together grow
- The pressure inside – and hence the temperature - grows
- Eventually the atoms of Hydrogen start to fuse together to form Helium
- This fusion process is exothermic

The Life of a Star

- A balance is achieved between the gravitational forces attempting to squeeze the star inwards and the pressure produced by the heat generated within the star.
- A star in this stage of its life is remarkably stable it is in its "Main Sequence"
- Our own sun has been essentially stable for the last 4.5 billion years and is only half way through its life.
- A star dies when it runs out of Hydrogen

The Death of a Star

- The process of star death depends almost entirely on its mass
- We measure a star's mass in terms of the number of times heavier it is than the Sun
- The mass of the Sun is 1.989×10^{30} Kg
- Its surface temperature is 5,778 deg K

The Death of our Sun

- Our sun is getting hotter by about 1% every million years
- When the Hydrogen in the core is consumed the fusion reaction takes place in the outer regions
- The Sun will firstly expand to become a Red Giant
- Then collapse to be a White Dwarf

The Death of a Star

- Stars range in size from 0.1 to 100 solar masses
- They range in surface temperature from 2,500 deg K to 30,000 deg K
- Note: 0 deg K equals -273 deg C

Stars < 1.4 Solar Masses

- Less than the Chandrasekhar Limit
- Expand to become Red Giants before collapsing to become White Dwarfs then cooling down
- Subrahmahyan Chandrasekhar (aged 19) on the boat to the UK to work under Sir Arthur Eddington
- Shared the 1983 Nobel Prize with Willie Fowler. The scandalous omission of Fred Hoyle

1.4 < Stars < 10 Solar Masses

- Collapse after a supernova to become Neutron stars
- Extremely dense
- Typically 2 solar masses with a diameter of 11 km
- One teaspoon full would weigh 20 billion tonnes
- They spin rapidly up to 1,400 times a second
- Pulsars as discovered by Jocelyn Bell (1967) – yet another Nobel scandal
- Little Green Men!

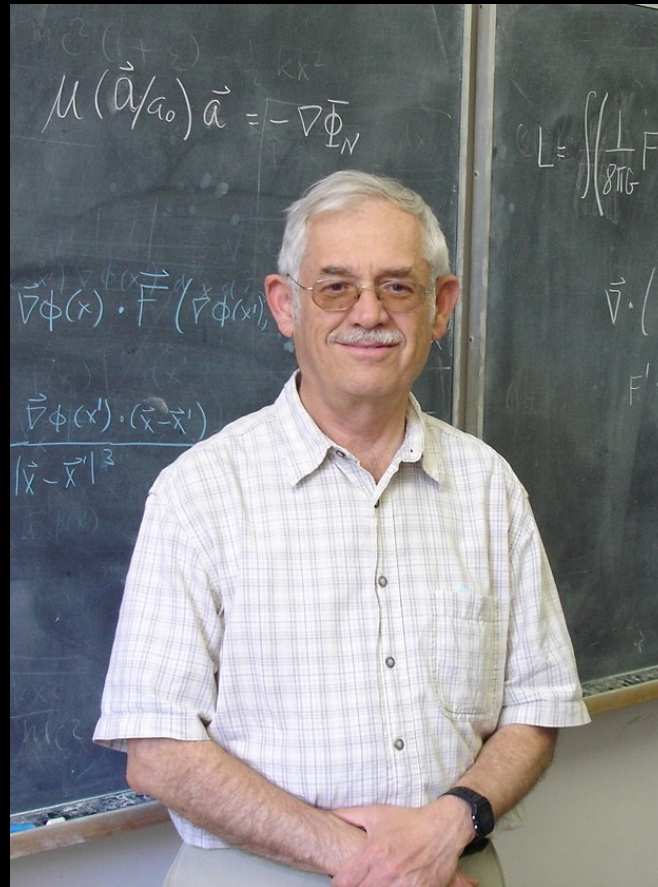
Stars $>$ 10 Solar Masses

- Collapse into Black Holes
- The Landau – Oppenheimer – Volkoff Limit
- Lev Landau 1962 Nobel Prize
- A serious car accident in January 1962 prevented him from going to Stockholm. Medical teams from US, Canada, France, Czechoslovakia came to help.
- These large stars continue to collapse beyond the Schwarzschild Radius and become Black Holes

Stars $>$ 10 Solar Masses

- Black Holes
- But this is not the end of the story....

Jacob Bekenstein (1947 - 2015)



Jacob Bekenstein

(1947 - 2015)

- In 1972 he suggested that there should be a Thermodynamics of Black Holes
- That they should have associated with them both Entropy and Temperature

Jacob Bekenstein

(1947 - 2015)

- At first these ideas were widely ridiculed
- Not least by one of my old friends

Stephen Hawking

(1942 – 2018)



Stephen Hawking

(1942 – 2018)

- But Stephen changed his mind
- Embraced the idea of Black Hole Thermodynamics
- In 1974 proposed a mechanism by which they could radiate: Hawking Radiation

Hawking Radiation

- Empty space ain't empty
- Virtual particles continually pop into existence
- They have opposite charge but identical mass

Hawking Radiation

- When
- Virtual particles occur near the Event Horizon
- One particle may be captured by the Black Hole and the other escape!

Hawking Radiation

- The flux of escaping particles is the Hawking radiation
- Since they have positive energy
- Their swallowed partners **MUST** have negative energy

Hawking Radiation

- Negative energy is tantamount to negative mass
- Recall the Energy equivalent of Mass
- $E=mc^2$
- Thus

Hawking Radiation

- The Black Hole is losing mass
- It is **Evaporating**

Black Hole Evaporation

- Extensive calculations have led Hawking to show that the temperature of a Black Hole is inversely proportional to mass
- Hawking's formula is one of the triumphs of modern physics

Temperature of a Black Hole

$$T = \frac{hc^3}{16\pi^2 kGM}$$

Temperature of a Black Hole

- h = Planck's Constant
 - 6.62×10^{-34} Joules

 - C = Velocity of Light
 - 2.99×10^8 metres/sec

 - $\pi = 3.1416$

 - K = Boltzmann's Constant
 - 1.38×10^{-23} Joules per Kelvin
- G = Newton's Gravitational Constant
 6.67×10^{-11} Newton m^2 per $kgms^2$

Temperature of a Black Hole

$$T = \frac{hc^3}{16\pi^2 kGM} = \frac{6 \times 10^{-8}}{M_s}$$

Reduced Planck's Constant \hbar

$$\hbar = \frac{h}{2\pi}$$

Westminster Abbey



Temperature of a Black Hole



Black Hole Evaporation

- The smaller ones are hotter
- In fact only very small ones are hotter than space!

Black Hole Evaporation

- The rate of evaporation is determined by the difference between the Hawking temperature and the temperature of surrounding space
- We now know the temperature of space it is 2.725 K

Black Hole Evaporation

- Since the temperature of space is 2.725 K
- We can calculate the mass of a Black Hole of this temperature:
- $6 \times 10^{-8} / 2.7 = 2.22 \times 10^{-8}$ Solar Masses
- Since the solar mass is 1.988×10^{30} Kgms
- We get 4.44×10^{22} Kgms

Black Hole Evaporation

- The answer was 4.44×10^{22} Kgms
- Almost exactly the mass of the Moon
- 4.5×10^{22} Kgms
- Question: what would be the diameter of such a Black Hole
- Answer 13×10^{-6} m
- And the diameter of a human hair?
- Between 17 and 181×10^{-6} m

Hawking Temperature

mass	Schwarzschild radius	temperature
solar mass	3 kilometres (1.9 miles)	1 tenth of a millionth Kelvin
mass of the earth	9 millimetres	0.02 Kelvin
mass of the moon	1/10 millimetres	1.7 Kelvin
1/10 mass of the moon	1/100 millimeter	17 Kelvin
1/100 mass of the moon	1 millionth of a metre	170 Kelvin
1/1000 mass of the moon	1/10 millionth of a metre	1700 Kelvin
1/2000 mass of the moon	1/20 millionth of a metre	3300 Kelvin
1/5000 mass of the moon	1/50 millionth of a metre	8400 Kelvin

Black Hole Evaporation

- Hawking radiation has yet to be detected
- Sadly both Jacob Bekenstein and Stephan have died so neither will get, or share, the Nobel Prize

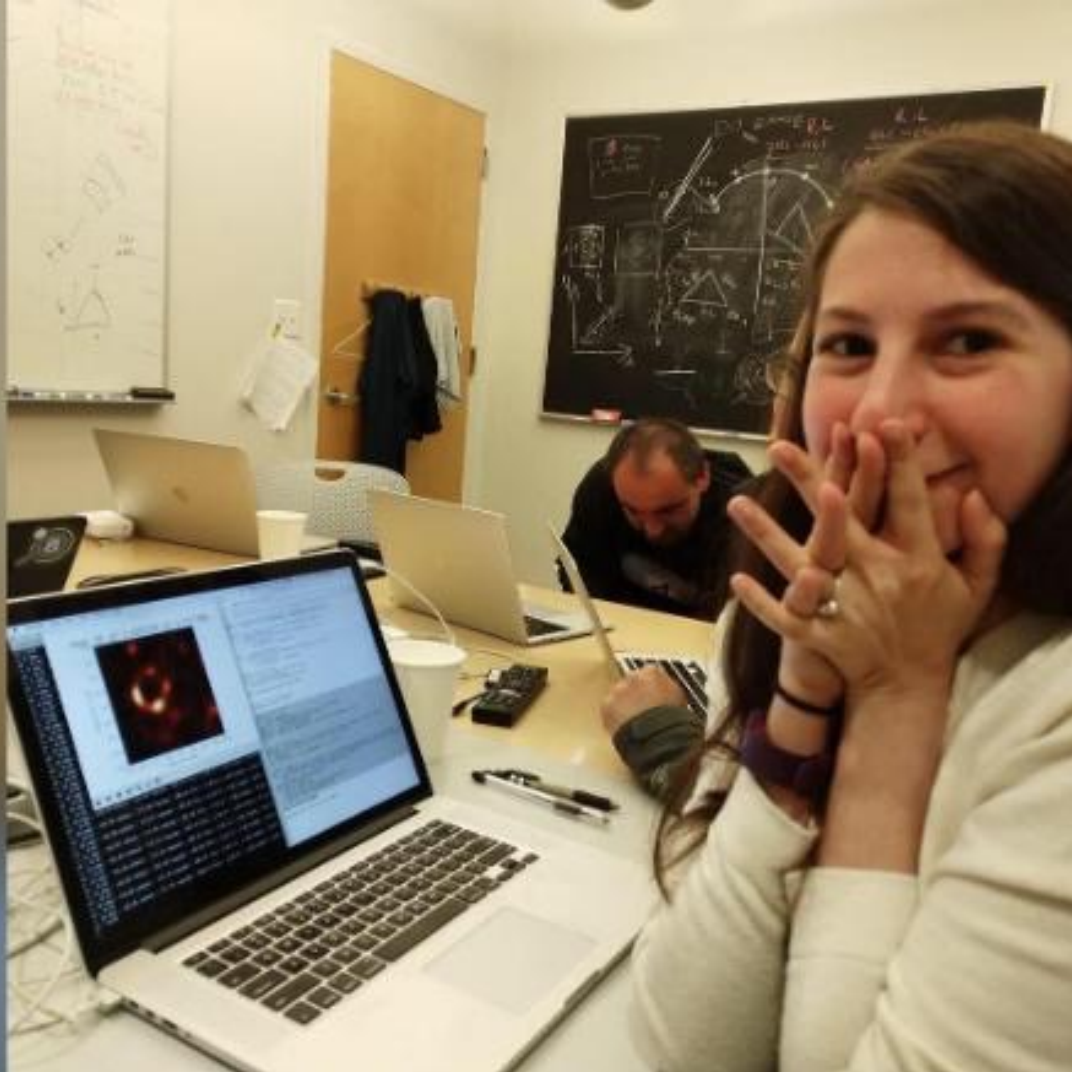
Gravity Waves

- 2016 detected
- Coalescing Black Holes
- Data lasting two tenths of a second
- Frequency chirp from 15 Hz to 75 Hz

Gravity Waves

- 2017 detected
- Coalescing Neutron Stars
- <https://www.ligo.caltech.edu/video/ligo20171016v8>

Katie Bourman (1990 -)



Event Horizon Telescope

- Aperture synthesising telescope
- Eight ground-based radio telescopes
- Hawaii, Arizona, Spain, Mexico, Chile, and the South Pole

Event Horizon Telescope

- Messier 87, a massive galaxy in the nearby Virgo galaxy cluster. This black hole resides 55 million light-years from Earth and has a mass 6.5 billion times that of the Sun.

The image of a Black Hole

- https://www.youtube.com/watch?v=S_GVbuddri8&ab_channel=Veritasium

The image of a Black Hole

Happy Albert



The End